



RHS Garden  
Harlow Carr

# Mathematical problem solving trail

[2012 version]

Name: \_\_\_\_\_

School: \_\_\_\_\_



### 34. Elliptical beds

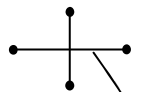
(49,10)

*Make sure you have completed 'Make your Bed' before trying this activity*

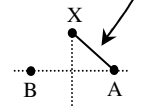
The elliptical flower bed was pegged out by the gardeners using measuring tapes and poles. Follow these instructions to mark out your own smaller flower bed.

a) Decide how long and how wide you want your flower bed to be.

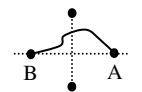
b) Lay out two tapes to match your chosen length and width. These will be the lines of symmetry. They are the *major* and *minor* axes of the ellipse. How will you be sure that they bisect each other at right angles? Put in some poles to mark the ends of the tapes.



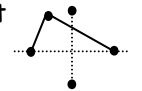
c) Now calculate half the length of your ellipse. Hold a tape at X and measure this distance to find point A. Repeat for B and put in two poles at points A and B. Each of these points is a *focus* of the ellipse.



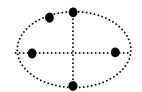
d) Now hold a measuring tape at points A and B. Set the length to exactly the same as the length of your ellipse (the major axis).



e) Pull the tape tight into a triangle - make sure that two people keep it held at A and B. Put in another pole to mark the third vertex of the triangle.



f) Repeat for other triangles, always held at poles A and B.



### Welcome to the Harlow Carr Problem Solving Trail

We hope that you enjoy working through these problems. They are all designed to make you think, so don't worry if you can't answer them straight away. Sometimes there are no right answers, and there may be several different ways of tackling a problem. Discuss your ideas with a partner. Which is likely to be the best way?

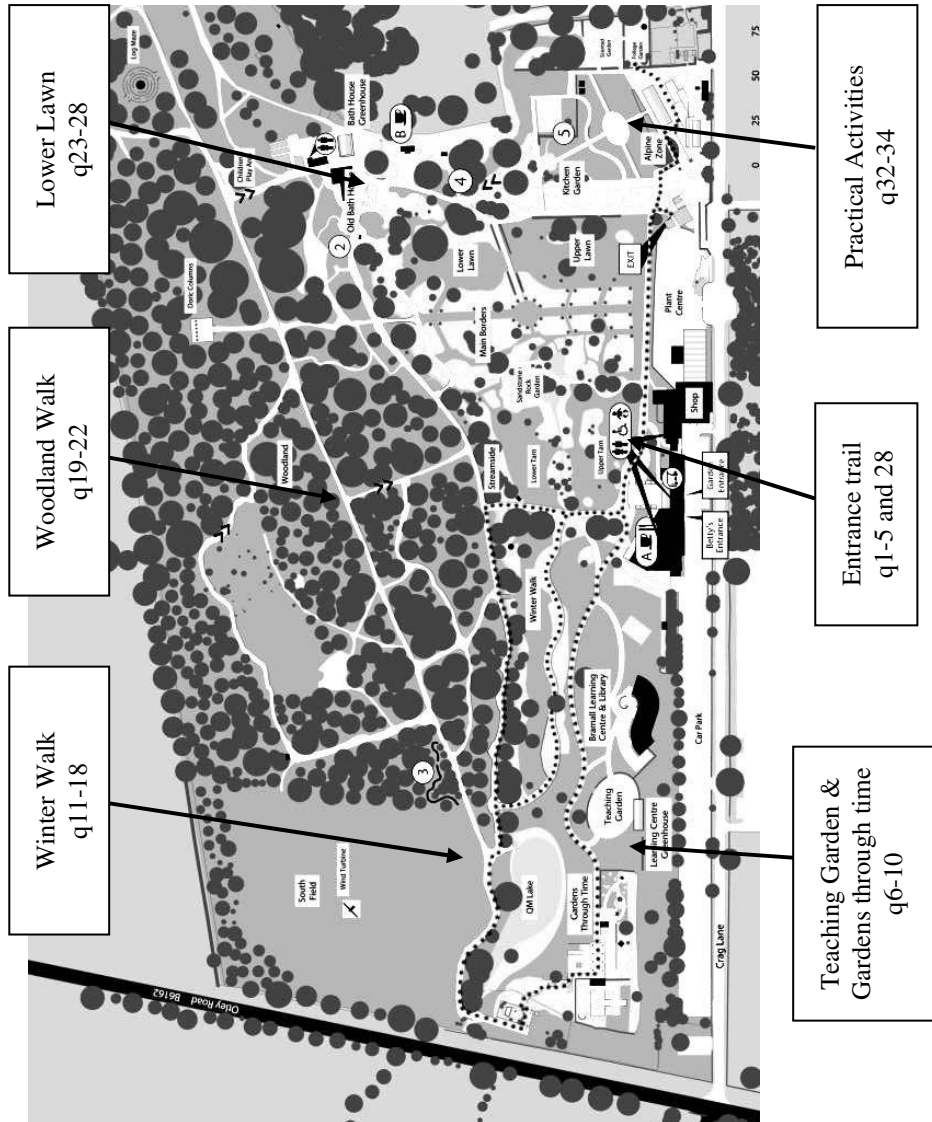
There will not be enough time to answer all of the questions. Your teacher might suggest which ones to try, and the order in which to do them. Use the space below to write down which ones you will be doing.

I am going to try these questions:

I will do the practical activities (q 32 - 34) at \_\_\_\_\_ am / pm

In an emergency I will find a teacher at

(draw the position on the map below)



### 33. Make your bed ...

(48,8)

*Make sure you have completed Ropey Shapes before trying this activity*

The gardeners use measuring tapes and canes when they are marking out a rectangular flower bed. They don't use any tools to measure the right angles.

P) How can three measuring tapes be used to measure an exact right angle?

F) Mark out a rectangular garden that is 3m wide and 4m long

F) Now make a square bed that is exactly 2m x 2m

## 32. Ropey shapes

(50,9)

You need a marked loop of rope and some canes for this activity.

P) Use your rope to make these shapes.  
Sketch each one and write down the lengths of the sides.



### A rectangle

How many different rectangles can be made?

### A square

### A regular hexagon

What if it doesn't have to be regular?

### An equilateral triangle

### An isosceles triangle

How many different ones can be made? How do you know?

### A right angled triangle

\*\*\* remember how to make a right angled triangle \*\*\*

## 1. Hand rail

(29,8)

F) How would you describe the curved shape that supports the handrail?

C) Does this shape have a mathematical name?

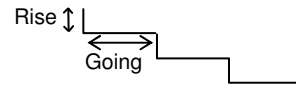
P) An architect wants to use a similar handrail for another project. Sketch the shape and record any measurements that will be needed to make an exact copy.

D) Construct an accurate scale drawing of this shape

## 2. The entrance steps

(29,8)

The height and depth of a step are called the 'rise' and the 'going'.



According to the building regulations for houses:

- the sum of the going and twice the rise must be between 550mm and 700mm.
- The rise must be between 75mm and 170mm.
- The going must be at least 250mm.

C) Write down three inequalities to show this information

F) Does the flight of steps meet these regulations? If yes, how do you know? If no, why do you think it is allowed?

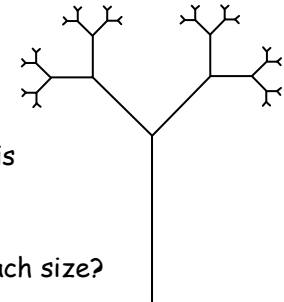
F) What is the gradient of each flight of steps?

P) The maximum pitch of a flight of steps is  $34^\circ$ . How much higher could the rises be to stay within the legal maximum?

## 31. Fractal tree

(?,?)

Look at this tree. At the top of the trunk there is a branch going off at  $45^\circ$  that is half as long as the trunk. Follow this branch. At the end of it there is a branch going off at  $45^\circ$  that is half as long .....



C) How many mini trees are there of each size?

Describe the pattern.

Find a formula.

P) Sketch a fractal tree based on different proportions or angles.

D) Try changing the angle of the branches.

D) Can you draw a fractal poplar tree? Oak tree?

### 30. What shape is that tree?

(?,?)

Find a tree where you can clearly see the shape of the branches. In many species of trees, the branches all come off the main trunk at roughly the same angle.

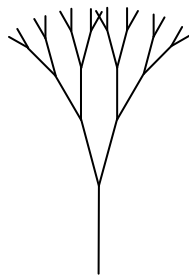
C) Look at the first main branch that comes off the trunk. Estimate the angle between the branch and the trunk

C) At roughly what fraction of the height of the whole tree is the first branch? (eg half way up? ... three quarters? ... one third?)

C) Now look along one of the main branches. Does it split at roughly the same fraction into another branch?

On some trees, each branch has similar proportions and angles to the full tree. This self-replicating structure is an example of a *fractal*. Look at the picture to see how this works.

P) Is your tree a fractal tree? Draw a sketch to show the main branches of your tree.



### 3. The long and winding path

(28, 8)

Stand on the steps and look at the network of paths to your left. They are shown on this map:



F) The paths need to be checked each day, starting in the Teaching Garden. Which is the best route to take so that all the paths are followed? Draw the best one on the map.

D) Draw some other simple networks of paths. Which of them can be drawn without taking the pencil off the paper or retracing any paths? What advice would you give someone who is trying to decide where to start?

#### 4. Best laid plans

(17,8)

F) Look at the plan view (birds' eye view) of this building on your map. Use mathematical words to describe its shape accurately.

F) Walk to the path at (16,12). Sketch the elevation (front view) of the building from this point.

P) Look at this curved gutter. Where will the rainwater flow fastest?



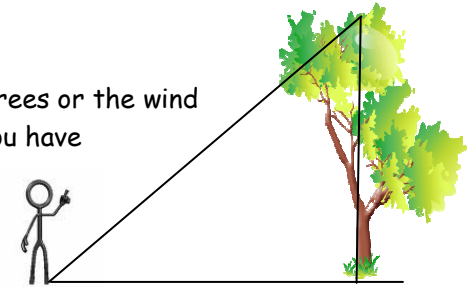
Draw arrows at different points on the photo to show the speed of flow - use short arrows for slow, longer arrows for fast.

C) The curves on the roof are similar to the graph of a mathematical function. Which type of function might it be?

D) Load a photo of the roof onto your computer. Use graph sketching software to find a function that matches it as closely as possible

#### 29. How high the tree (4) (6,15) or (18,11) or (43,11)

Go back to one of the trees or the wind turbine whose height you have already estimated.



C) What angles or lengths can you measure on this diagram?

F) Use trigonometry to calculate the height of the tree.

P) Would the calculation be more or less accurate if you stood further away from the tree? Why do you think this?



28. Make time, save time, while time lasts (36, 10)

Use the sundial to work out which direction is North on your map.

C) In which direction is the clock from here?

F) Does the sundial tell the right time? If not, what might be the reasons for this? How could it be adjusted?

F) In which direction does a person walk if they follow the long path from (45,31) to (10,17)?

C) What is the equation of this path?

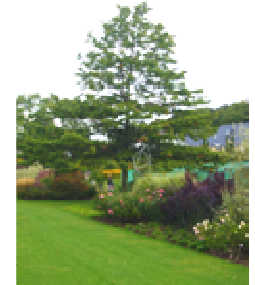
5. How high the tree (1)

(18,11)

Ask a friend to stand beside this tall tree.

C) How tall is your friend?

F) Describe how you could use the height of your friend to estimate the height of the tree.



P) Approximately how high is the tree?

F) Two students were arguing about whether it is more accurate to use a very tall friend or a very short friend. What do you think?

## 6. What a pane

(12, 9)

C) How many glass windows are there in this greenhouse?



P) Write down some different ways of calculating the answer.

The greenhouse is built in sections (look at the roof to see the joins between them). It has five complete sections, and one part section at the end. Each section has different types of windows: small fixed, small opening, large fixed and roof windows.

P) How many of each type of window will there be in a greenhouse with three complete sections?

With nine sections?

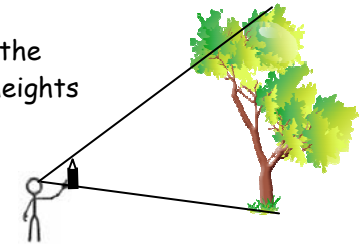
P) Some people might have multiplied by three to get this answer. Why doesn't this give the correct answer?

D) Make a formula to give the number of windows for any sized greenhouse

## 27. How high the tree (3)

(43, 11) or (44,7)

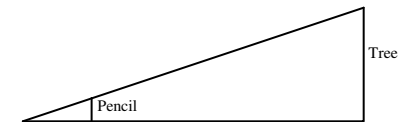
In this question we find out how the Harlow gardeners estimate the heights of trees.



Stand in front of a tall tree and hold a pencil pointing upwards at arm's length in front of you.

Move the pencil towards your eye until it just covers the height of the tree.

Look at these triangles:



C) What lengths do you know?

C) Why are the triangles similar?

P) Estimate the height of tree

## 26. Raindrops keep falling...

(50,7)

Harlow Carr has an annual rainfall of approximately 800mm.

The rain that falls on the roof of this building is collected and stored in a rainwater harvesting tank. It is then used to water the plants inside the house.

C) Estimate the length, height and width of this building.

F) Estimate the floor area of the building.

P) Calculate the volume of water that is collected each year (in  $\text{m}^3$ ). How many litres is this?

F) What assumptions did you make when working this out?

## 7. Getting warmer

(11, 9)

F) The thermometer scale shows that  $10^\circ\text{C} \approx 50^\circ\text{F}$  and that  $60^\circ\text{C} \approx 140^\circ\text{F}$

Sally thought that  $60^\circ\text{C}$  should be six times warmer than  $10^\circ\text{C}$  but that would be  $300^\circ\text{C}$ .

Can you explain why this is not correct?



P) What temperature is the same as  $110^\circ\text{C}$ ?

F) How many  $^\circ\text{F}$  is each  $^\circ\text{C}$ ?

F) Work out a rule to change any temperature from  $^\circ\text{C}$  to  $^\circ\text{F}$ . Use the thermometer scale to check that your rule works.

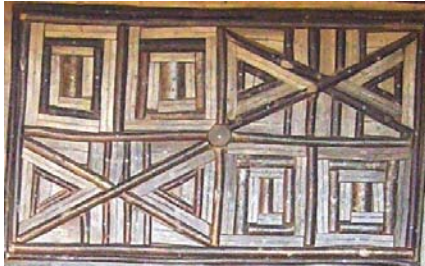
F) Write your rule as a formula.

D) Find out how to convert  $^\circ\text{C}$  to  $^\circ\text{F}$ . Is this the same as your method? What about converting from  $^\circ\text{F}$  to  $^\circ\text{C}$ ?

### 8. Just a half?

(7,9)

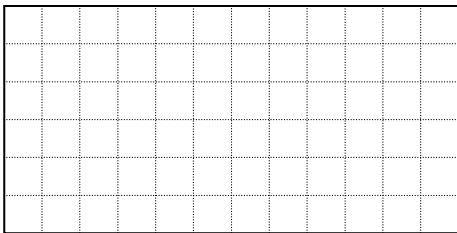
The back wall of this garden house has a panel made from light and dark wood.



C) Does it contain more light or more dark wood?

F) Look at one section of the panel. What fraction do you estimate is dark wood? Why do you think this?

D/P) Design a panel on the grid below that has exactly equal areas of light and dark wood.



### 25. Raised beds

(47,12)

Find one of the large raised beds in the kitchen garden.

F) What is the volume of soil in the bed? How many  $\text{m}^3$  is the volume? How many  $\text{cm}^3$ ?

C) How many 60litre sacks of compost are needed to fill each bed? [At Harlow Carr they make their own compost - go and have a look how while you are in the Kitchen Garden]

F) The edging strips are made from recycled milk bottles. Each large bed uses 880 bottles. What is the total volume of the four plastic strips surrounding the bed? Estimate the volume of plastic that is used to make a single milk bottle.

## 24. Crazy paving

(41, 15)

C) This path is made from bricks and paving slabs. What different shapes are they?



C) Two of the shapes have angles that are not right-angles. Sketch these, and calculate the size of their angles.

F) What were the clues that helped you to calculate these angles?

P) How many of each shape are there in the path?

## 9. The bench with the hole

(6,9)

The shape of this bench is an *annulus* - a circle with a hole cut in it.



F) Which has the larger area, the hole or the wooden annulus?  
(Explain how you know)

P) What would be the radius of the hole if the area of the wood and the hole were exactly equal?

### 10. Red pagoda

(4,9)

Each layer of this structure is rotated and enlarged to make the next layer down.

C) What is the angle of rotation between each layer and the one that it rests on?

F) Sketch the plan view (from above) of this building, showing how the different layers fit together.



C) Compare the heights of each layer.  
Estimate the scale factor of the enlargement at each stage.

P) How tall would the building be if it carried on growing this way indefinitely?

### 23. As old as the trees?

(47,18)

The gardeners use a formula to estimate the age of a tree:

$$\text{Age in years} = \frac{\text{circumference in cm} \times 4}{10}$$

C) Find the cedar tree in this photo (it has bluish green leaves). Measure its circumference about 1.5m above the ground.

Use the formula to estimate its age.



DP) The formula works by estimating the number of rings that the tree has. How wide does this assume that each ring is?  
[don't spend too long trying to work this out in the Garden]

P) What other assumptions does this formula make?

## 22. What rocks

(38,20)

F) Imagine the memorial stone is boxed in by a cube. Estimate what fraction of the cube the stone takes up.



F) Estimate the volume of the memorial stone.

C) The density of sandstone is about  $2000 \text{ Kg/m}^3$ . What is the mass of the memorial stone?

P) The world record for a 'snatch' lift is 214 Kgs. Could an Olympic weight lifter snatch up this block? If not, how many weight lifters would it take?

## 11. How high the tree (2)

(6,15)

Find this tree which grows up through a curved bench.

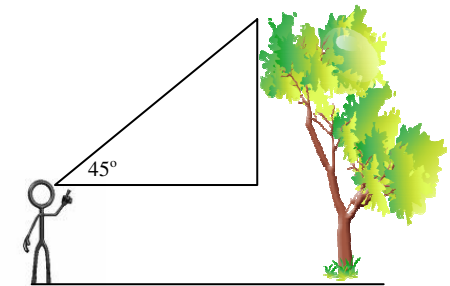


C) What mathematical symbol does the shape of the bench remind you of?

Find a place to stand so that the top of the tree is at an angle of  $45^\circ$  from your eye.

C) What type of triangle is shown on this diagram?

F) How high is the tree?



P) Why might different people have different answers to this question? What advice would you give them?

## 12. It's a breeze

(6,24)

If the sun is shining ...

C) Stand beside the turbine. How long are the shadows? You:                      Turbine:



F) Use this information to estimate the height of the tower.

If it is windy ...

F) Estimate how many times the blades turn in a minute

C) The blades are 4.5m long. How far does the tip of a blade travel in each revolution?

F) How fast are the tips of the blades travelling in km/hr?

Whatever the weather ...

When running at full speed the turbine has an output of 15kW.

D) Do you think that this is sufficient to provide all of the power to the Bramall Learning Centre? Why?

## 21. Busy bees

(42,24)

Be very gentle when opening the screens to look inside the beehive. Remember to shut the screens when you leave to help the bees conserve heat. The bees will not be here if they have been moved to their warmer site during the winter.

P) Think of three different ways to estimate the number of bees in the beehive.

F) How many bees do you think that there are in this hive?

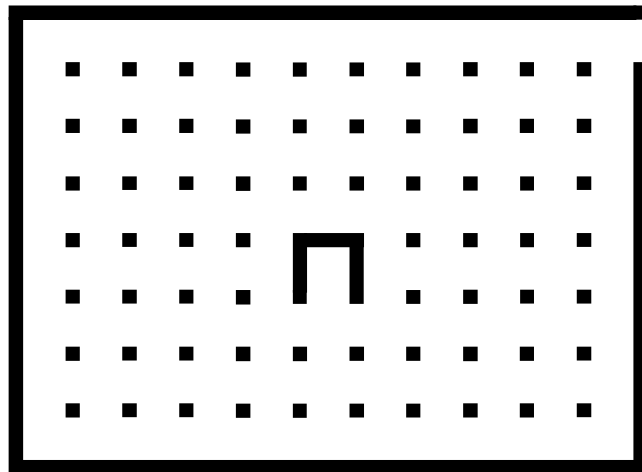


## 20. Be amazed

(53,34)

F) Sketch a plan of the paths inside the log maze

P) The log maze has lots of dead ends, but just one path to the centre. Design your own rectangular maze on the grid below with a single path to its centre.



D) Find out about the history and mathematics of mazes and labyrinths

## 13. Crop trials

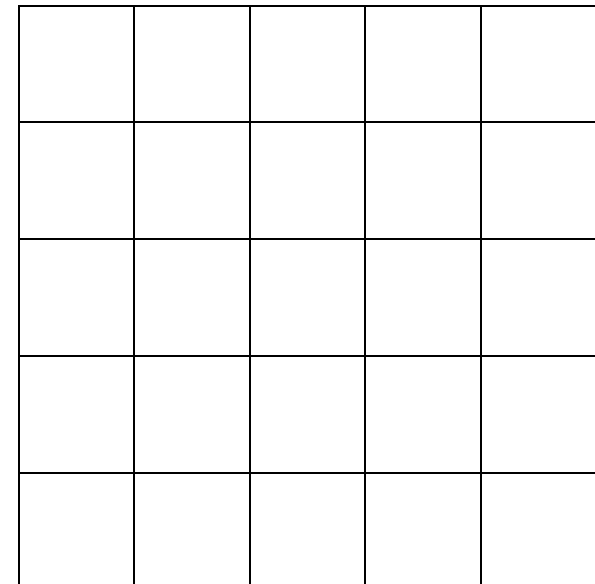
(10,18)

This part of the garden is sometimes used for crop trials.

When comparing different seeds or fertilisers it is important that each is tried under the same conditions. Lots of factors can make a difference: the direction the ground faces, the amount it slopes, the type of soil, etc. These are evened out by making sure each seed is trialed under the same combination of conditions. This is done by making a grid of trial plots.

P) Plant crops A, B, C, D and E on the plot below, one type per square, so that each type appears in each row and each column

P) Now add fertilisers 1, 2, 3, 4 and 5 so that every fertilizer appears in every row, every column and with every crop once.



14. If you go down to the woods today... (14,27)

Find the log grotto with the living fern roof.

Go inside and look up.



C) Estimate the total area of the sloping part of the roof.

F) When it is wet the roof weighs up to  $110\text{kg/m}^2$ .  
What is the maximum weight that the shed will have to support?

P) How many students is this? (don't try this out!)

19. It's all Greek

(37, 32)

P) Estimate the height of these columns.



F) Why did you choose to do it this way?

F) The columns are not cylindrical. They get narrower at the top. Why do you think this is?

The shape of each column (ignoring the fluting) is a *frustum* of a cone (a cone with the top cut off).



P) How tall would the columns be if the builders had carried on upwards until they had made a complete cone?

## 18. Set theory

(17,13)

The bench is surrounded by a circular paved area.

C) Estimate (or count) the number of stones (or sets) in the smallest circle.

P) Devise a rough formula for the number of stones in each circle.

F) Sketch a graph to show how the number of stones in the circles increases.



C) What sort of relationship is this?

## 15. Monstrous

(14, 19)

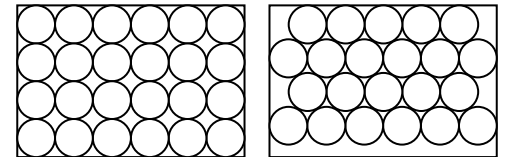
C) Look at a single log in the body of the Log Ness monster.  
How many other logs touch it?



F) If all the logs had exactly the same diameter they would stack together in a hexagonal array. Why is this?

F) Identical logs can be packed into boxes in different ways:

Which do you think is the most efficient way of packing? Why?



Will this always be the most efficient?

### 16. The giant gardener

(19,16)

Find the giant spade.

C) Estimate the scale of the sculpture.



F) How long would a sculpture of a slug be if it were made to the same scale?

P) How tall is the gardener who would use these tools?

D) What size boots would the gardener take?

PD) How large could a person grow before their leg bones are no longer able to support their weight?

### 17. A bench with a view

(17,13)

Eight people can sit on this bench, each facing in a different direction.

C) What is the angle between each of the directions that people would face?

F) The photograph shows part of the seat. Use what you know about the shape of the bench to work out each angle that the joiner would have cut when it was made.

