RHS Garden Harlow Carr

Mathematical problem solving trail

[2012 version]

Teachers' Notes

North Yorkshire County Council
Children and Young People's Service

CITY OF YORK COUNCIL

Harlow Carr Mathematical Problem Solving Trail

What is the problem solving trail?
A collection of puzzles and problems designed to help students discover the 'real life' mathematics that is all around them as they visit the Garden.

There are five separate documents:

- Problem solving trail booklet – the full trail of 34 problems
- KS3 Trail booklet – based on the full trail but with fewer questions and reduced or more accessible content
- Scavenger Hunt – a series of mathematical objects to look for in the Garden. Can be followed separately or at same time as the trail.
- Map, with coordinates to locate each activity – essential to have this when following either trail
- Teachers' notes – this document

Why do the trail?
The trail encourages group work and collaborative problem solving. It develops observation and recording skills and reveals mathematical structure that might otherwise go unnoticed. It embeds mathematics in a context and provides opportunity for genuine functional application. The trail offers a set of uncontrived contextual examples that support and strengthen students' understanding of Mathematical Processes and Applications across Key Stages 3 and 4.

How long does it take?
There is too much material to meaningfully cover in a day. Schools have previously used the trail in half or full day blocks, but have pre-selected the questions that are to be tackled. The Scavenger Hunt can be followed in less time (anything from an hour upwards) but this will not give students a broad, rich experience of problem solving and practical mathematics unless they also complete at least some of the full trail.

What's the difference between the KS3 and the full trail?
The KS3 trail is made up from selected problems taken from the full trail. These are usually more accessible or require lower level mathematics in order to solve them. In some cases they have been rewritten or modified to suit a KS3 context. Question numbers are the same as those in the full trail, so it may look as if some questions are missing – they're not!

Which questions should I use if there is not time to follow the full trail?
This depends on what you want students to gain from their trip to Harlow Carr.
The table on the next page shows the topics covered in each question so that an appropriate selection can be planned in advance. The selection could be based around topic (eg trigonometry and geometry, sequences) or difficulty or location. Alternatively, different groups could follow mini-trails in different parts of the Garden and then share their findings after the visit. The mini-trails are: Entrance (q1-5 & 28), Teaching Garden & Gardens Through Time (q 6-10), Winter Walk (q 11-18), Woodland Walk (q 19-22), Lower Lawn (q 23-28) and Practical Activities (q 32-34). These can be followed independently in any order.
There is also a scavenger hunt that can be used independently.

Questions are classified as:

C: content questions, usually recalling facts or applying prior knowledge (GCSE AO1)
F: functional questions, tend to be more contextual and do not necessarily have a single correct answer ('Choosing and Using' the mathematics – GCSE AO2)
P: problem solving questions, usually multi-stage requiring interpretation of the question and planning a solution (GCSE AO3)
D: development questions, to be followed up after the visit

The D questions are to follow up after the visit: remind your students that they should not be attempted on the day.

Solutions

No solutions are provided. This is because many questions invite students to select their own objects to measure, draw, etc. More importantly, the questions are intended to strengthen students' problem solving and process skills, and so the way in which they have approached a problem and the mathematical methods that they have selected are more important than the final answer. There are many possible routes to the solutions: students should be encouraged to discuss possible methods in their groups before selecting the most appropriate. Teachers might also discuss the efficiency of different methods while students are working on the problems and encourage them to use more sophisticated mathematical techniques when appropriate. After the trail has been completed it would be helpful for students to compare their approaches with other groups and consider the effect that these had on their solutions.

What preparation needs to be made before the visit?

- Information about school visits to the garden and specimen risk assessment forms are available to download from: http://www.rhs.org.uk/Children/For-schools/School-visits/Harlow-Carr
- The problems are designed to be solved collaboratively. It will be useful to allocate students to groups of three or four before the visit. If your students are not used to working as groups they will get more benefit from the visit if they do some preparatory problem solving activities before they arrive. Some groups may benefit from working with an adult or other mentor (perhaps a TA or older student?).
- It will be helpful if students know their own weight and height before the visit.
- Preparatory work on maps and coordinates will help students to follow the trail more easily. **Students will gain more from their visit if they have marked the location of the different problems on the maps and planned their routes before they visit the garden.** These can be recorded on p2 of their booklets. Laminated maps will be provided on the day, but these do not show the locations of the activities.
  NB If a location lies inside a square on the map, the coordinates of the bottom left corner of the square are given
- Students should be familiar with the content of the *Scavenger Hunt* so that they know what to look out for before they set off around the garden
- Several of the activities require the use of clinometers. These are provided on the day, but students may wish to practise before the visit if they haven't used them before. Or they could make their own to bring along!
  (see http://nrich.maths.org/5382 or http://www.subtangent.com/maths/resources/clinometer.pdf)

What happens after the visit?

Many of the questions are intended to be followed up when students return to school. As well as using the trail as a stimulus to explore more mathematics, students should be given opportunities to discuss and compare their different approaches to solving the problems. This could be done through preparing group presentations on the sections of the trail that they have covered. They could also design their own local trails.

Equipment:

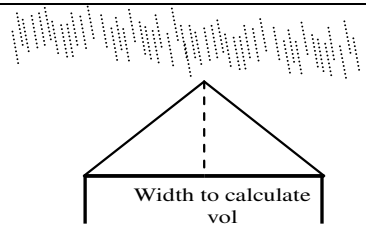
- Each group of three or four students will need a coordinate map of the garden, a calculator (scientific for some questions), a 20m measuring tape and a clinometer.
- The maps, tapes and clinometers are available from the Learning Centre in the Garden. Students should bring their own calculators.
- Each student should also have a pencil / pen and some rough note paper.
- Questions 32, 33 and 34 require sets of bamboo canes, shorter measuring tapes and some prepared loops of rope. These are all available from the Bramall Learning Centre.
- Cameras are desirable but not essential. Students will be able to record what they have done, gather facts for the development work at school, and be able to produce better presentations when reporting back to other groups.

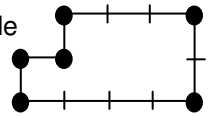
Question content

Question	No.	Topics	Equipment	Notes
Hand rail Full trail only	1	Shape Scale drawing Ratio / proportion Symmetry Construction	Measuring tape	Designed to strengthen observation skills and use of precise mathematical language. For the D question students may need to discover how to find the centre and radius given an arc of a circle.
The entrance steps Full trail Modified in KS3 Trail	2	Measurement Ordering Substituting into formulae Gradient (C) Trigonometry (P)	Measuring tape Scientific calculator	The P question is more efficiently answered using trigonometry but students may well devise their own methods that don't require trig, for instance scale drawing or direct measurement. KS3 Trail doesn't include the trig question.
The long and winding path Full trail KS3 Trail	3	Traversable networks Topology		Traversability (especially the Königsberg Bridge problem) appears in most recreational maths books. Lesson ideas are available at nrich: http://nrich.maths.org/2326 , http://nrich.maths.org/2327 (interactive exploration), http://nrich.maths.org/2484 Also see: http://www.suffolkmaths.co.uk/pages/Maths%20Projects/Projects/Topology%20and%20Graph%20Theory/Traversable%20Worksheet.pdf
Best laid plans Full trail Modified in KS3 Trail	4	2D representation Plans, elevations Gradient (& vectors) Functions and graphs Transformation of graphs		Students may need support in recognising that 3D curves may appear as straight lines when represented in 2D. The P question hints at differentiation and vector fields! Several functions might be appropriate in the C question. These can be explored further using graphical calculators or a graphing software package (D question). The function questions do not appear in the KS3 Trail
How high the tree (1) Full trail KS3 Trail	5	Estimation Proportion Scale factors	Measuring tape	This question encourages estimation of height through the use of proportion. Question 12 (*It's a breeze*) extends this to use similar triangles and proportion. Methods involving geometry and trigonometry are explored in other problems (*How high the tree (2), (3) and (4)).*
What a pane Full trail KS3 Trail	6	Counting Sequences Constructing formulae n^{th} term	Calculator	The C question is designed to encourage efficient methods of counting the window panes!
Getting warmer Full trail KS3 Trail	7	Temperature conversion Reading scales Proportional change Constructing formulae	Calculator	The F questions are intended to encourage students to build a formula that represents a generalised method, starting from specific numerical examples.

Question	No.	Topics	Equipment	Notes
Just a half Full trail KS3 Trail	8	Area Fractions		A film showing different ways of halving a square is available from the Association of Teachers of Mathematics at: http://www.atm.org.uk/resources/films/halves.swf
The bench with the hole Full trail KS3 Trail	9	Area of circle Solving equations (quadratic) Surds Enlargement Area scale factors	Calculator	A general solution to the P question requires the solution of $\pi(R^2 - r^2) = \pi r^2$, leading to $R=\sqrt{2}\, r$ Or it may be tackled through area scale factors and enlargement – the full bench is twice the area of the hole, so area SF of hole:bench is 1:2, giving length SF $1:\sqrt{2}$
Red pagoda Full trail KS3 Trail	10	Rotation Enlargement Plan views Sequences Limit of geometric progression Fractals	Calculator	No guidance has been given for the P question as this may spoil the sense of wonder as students discover for themselves that the sum of an infinite sequence may have a limit. As a follow-up activity students could explore other examples of fractal buildings, or construct their own. See http://classes.yale.edu/Fractals/Labs/BlockLab/BlockLab.html for activities using multilink cubes, or www.metacafe.com/watch/330851/paper_stairs_part_1/ for instructions on making fractal paper staircases. See q30 and 31 in the full trail for more examples of fractal constructions.
How high the tree (2) Full trail KS3 Trail	11	Notation (proportional sign) Measuring Isosceles triangles	Clinometer Measuring tape	See also *How high the tree (1), (3)* and *(4)* for other methods of estimating / calculating the height. Also *It's a breeze (q13)*
It's a breeze Full trail Modified in KS3 Trail	12	Ratio and proportion Similar triangles Angular velocity Circumference Compound units Conversion of metric units Estimation Angles around a point (KS3) Symmetry (KS3) Naming solids (KS3)	Measuring tape	See http://www.turbineservices.co.uk/wind-turbines/proven/ for further information on the turbine. The KS3 question omits the angular velocity and power consumption questions. It has extra questions on symmetry and shape. No guidance has been given on estimating the height from the shadow length. Students may need to be encouraged to sketch a pair of similar triangles. The D question is very open and is intended to spark an investigation into alternative energy sources and sustainability issues. Students will need to estimate energy consumption for the building – both heating and lighting, but they could be encouraged to go much further than this. The question could be the start of a much broader investigation tapping into many cross curricular links.
Crop trials Full trail KS3 Trail	13	Logic		Students may spot the link with Sudoku. What they are actually making is a *Latin Square.* When both crop type and fertiliser type are combined, this is a *Graeco Latin Square*. See interactive resource at http://www.cut-the-knot.org/arithmetic/latin.shtml
If you go down to the woods today Full trail KS3 Trail	14	Compound measures Estimation Area of rectangle		The P question compares standard and non-standard measures (the weight of a student) See also *What rocks (22), It's a breeze (12)* and *Raindrops keep falling (26)* for other questions involving compound measures

Question	No.	Topics	Equipment	Notes
Monstrous Full trail KS3 Trail	15	Tessellation Packing problems Area of circle, triangle Percentages Trigonometry		Triangular and rectangular packing arrangements will require area and trig calculations for full analysis of their efficiency - see lesson ideas at http://illuminations.nctm.org/LessonDetail.aspx?ID=L690 More straightforward area calculations are used to solve the examples given in the question, but these include an extra constraint in the size of the surrounding rectangle.
The giant gardener Full trail Modified in KS3 Trail	16	Measurement Estimation Ratio and proportion Scale factors and enlargement Similarity Graphs (D) Correlation (D) Functions (D)	Measuring tape	The graph and function work might arise through considering a graph of known foot lengths v boot sizes The P/D question does not appear in the KS3 trail. It encourages comparison of area and volume scale factors. The breaking stress of a human femur under compression is roughly 170 MPa ($\approx 1.7 \times 10^8$ N/m^2 $\approx 1.73 \times 10^7$kg/m^2) See http://www.askabiologist.org.uk/answers/viewtopic.php?id=6836 for a discussion of the ratio of mass to strength. This could also lead to discussion of surface area / volume ratios and how animals keep warm through huddling. Both these topics are discussed here: http://dinosaurtheory.com/scaling.html
A bench with a view Full trail KS3 Trail	17	Angles around a point and on a straight line Alternate angles, etc Polygons	None – especially no protractors	Students should calculate the angles on the bench seat from the information they know about the shape of the bench.
Set theory Full trail Modified in KS3 Trail	18	nth term Linear relationships Straight line graphs		Students may be surprised that the relationship is linear. Encourage them to explore why it is. What would be plotted to get the quadratic graph that might have been expected?
It's all Greek Full trail Modified in KS3 Trail	19	Estimation 3D Shape Proportion and similar shapes		Why do columns get narrower at the top? See http://romantech.wikispaces.com/A+-+Parthenon
Be amazed Full trail Modified in KS3 Trail	20	Mazes Topology		Mazes generate a lot of mathematics for students to explore. A good starting point may be: http://plus.maths.org/issue14/features/budd/index.html The KS trail does not suggest this further exploration.
Busy bees Full trail KS3 Trail	21	Estimation		The beehive is moved away from the garden during late autumn and winter.
What rocks Full trail KS3 Trail	22	Volume Estimation Density and compound measures	Calculator Measuring tape	

Question	No.	Topics	Equipment	Notes
As old as the trees Full trail Modified in KS3 Trail	23	Measuring Converting units Substitution Formulae Circumference of circle	Measuring tape	If there are n rings, each of width w, along the radius then r=nw, so C=2πnw. The age of the tree is the number of rings, so the formula can be rewritten as n=4C/10. Comparing n=4C/10 with C=2πnw gives n=8πnw/10, which simplifies to w=10/8π ≈ 0.4cm The KS question only covers measuring and substituting into the formula.
Crazy paving Full trail KS3 Trail	24	Tessellation Angle properties 2D shape Sequences n^{th} term	None – especially no protractors!	This could lead to a larger investigation of why some shapes tessellate. See nrich interactivity at http://nrich.maths.org/6069
Raised beds Full trail Modified in KS3 Trail	25	Measurement Estimation Volume of cuboid Metric conversion (m^3 to litres and m^3 to cm^3)	Measuring tape Calculator	
Raindrops keep falling Full trail KS3 Trail	26	Estimation Area of rectangle Trigonometry? Pythagoras? Volume of cuboid / prism Conversion between metric units	Calculator	Volume of water = area on which rain falls x 800mm Will students use the sloping roof area or the area of the horizontal projection of the roof (ie the floor area)? 
How high the tree (3) Full trail KS3 Trail	27	Measurement Similar triangles Proportion Enlargement	Pencil or pen – or a twig! Measuring tape Calculator	Compare this method of using similar triangles with q12 *It's a breeze*. See also *How high the tree (1), (2)* and *(4)* for other methods of estimating / calculating the height
Make time, save time Full trail Modified in KS3 Trail	28	Compass directions y=mx+c		There is a Bowland Mathematics case study covering sundials that might be useful for follow up work: http://www.bowlandmaths.org.uk/outline.htm#1 The y=mx+c question is only in the full trail.
How high the tree (4) Full trail only	29	Trigonometry Rounding errors	Scientific calculator Measuring tape	See also *How high the tree (1), (2)* and *(3)* for other methods of estimating / calculating the height
What shape is that tree Full trail only	30	Angle estimation Fractions Ratio Fractals		This activity is designed to encourage students to look more closely at the structure of trees. It can then be followed by the *Fractal Tree* question 31.

Question	No.	Topics	Equipment	Notes
Fractal tree Full trail only	31	Fractions Sequences Exponential Fractals		An internet search for 'fractal tree' will find many resources, some of which include downloadable generators. An iPhone app is available here; http://www.geom-e-tree.com/ Students who are inspired by this idea may like to explore how fractals are used to generate scenery and landscapes in computer games. See the interactivity at http://ibiblio.org/e-notes/3Dapp/Mount.htm
NB questions 32, 33 and 34 form a self-contained practical activity circus. It is recommended that a teacher stays in this area to lead the activities and to act as a contact point in case of emergency. Canes, ropes and measuring tapes are available from the Learning Centre. Ensure that all canes have rubber caps on.				
Ropey shapes Full trail KS3 Trail	32	Perimeter Area Shape Polygons Circles Pythagoras	Canes One x 12m loop of rope marked at metre intervals (or any length, but marked at 12 equal intervals)	This activity needs to be done before *Make your bed* and *Elliptical beds*. Students particularly need to complete the last question to establish that the 3-4-5 triangle is right angled. A nice follow up activity is to investigate all the shapes that can be made with a loop of rope that is 12 units long. Ie all have perimeter 12 when drawn on a squared grid. How many are there? Which has the largest area? What if vertices are not restricted to points on the grid? 
Make your bed Full trail KS3 Trail	33	Pythagoras Measuring Construction	Three x 10m measuring tapes Canes	Students need to know how to make a right angle from a 3-4-5 triangle. They should complete *Ropey shapes* before trying this activity.
Elliptical beds Full trail KS3 Trail	34	Measurement Construction Loci Pythagoras	Plenty of canes Three x 12m ropes Measuring tape	Students need to know how to make a right angle from a 3-4-5 triangle. They should complete *Ropey shapes* before trying this activity.

<u>Booking a visit</u>

School visits are free, but all parties must be pre-booked. Please contact :

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